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Four-Brane and Six-Brane Interactions in M(atrix) Theory

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Abstract We discuss the proposed description of configurations with four-branes and six-branes in m(atrix) theory. Computing the velocity dependent potential between these configurations and gravitons and membranes, we show that they agree with the short distance string results computed in type IIa string theory. Due to the "closeness" of these configuration to a supersymmetric configuration the m(atrix) theory reproduces the correct long distance behavior.

1 introduction

Recently [1] there has been a proposal for the microscopic description of M-theory [2, 3] in the infinite momentum frame. In this proposal the only degrees of freedom are the zero-branes and the lowest open string-modes stretching between them [1], giving an $SU(N)$ Yang-Mills theory [5]. In order to describe M-theory one has to recover its brane content [6, 7, 8], Lorentz invariance, long distance behavior, and correct compactifications. Compactification of m(atrix) theory were considered in [1, 9, 10, 11, 12, 13] and the long distance behavior of membranes was analyzed in [14, 15]. The description of the membrane was already given in [1], a description of an open membrane was given in [16] and a proposed description of the four-brane of type IIa (a wrapped five brane of M-theory) was given in [17]. The four-brane construction however involved introducing more degrees of freedom into the theory.

A different approach was suggested in [11, 18]. In [18] the supersymmetric algebra of zero-brane in the m(atrix) theory was analyzed, and it was shown that one has a conserved charge associated with the membrane and four-brane descriptions. They also showed how to construct in this frame work configurations with any dimensional brane.

Consider the membrane description in m(atrix) theory. The membrane is described through its effect on the zero-branes bound to it in a non threshold bound state. This can be seen [11, 15] by comparing the m(atrix) description to a type IIa description in which one takes a two-brane with a magnetic field on its world volume. Thus one can expect to be able to describe any brane in type IIa theory if it can be put in a non-threshold bound state with zero-branes. In the type IIa description, due to the coupling of the two-brane to RR background (A is the RR one-form gauge potential) $\int A \wedge \mathcal{F}$ [19], the zero-branes are taken into account by turning on a magnetic field on the two-brane. In order to bound a four-brane in a non-threshold bound state with zero-branes one can use the four brane coupling to A_μ, $\frac{1}{2}\int A \wedge \mathcal{F} \wedge \mathcal{F}$, with a constant magnetic field F. This also adds two-branes, through the coupling $\int C \wedge \mathcal{F}$, where C is the RR three-form gauge potential.

In the two-brane case the matrix description [1] was achieved by taking $[X_1, X_2] = Iic$, it is then natural to take for the four-brane, four matrices which satisfy $[X_1, X_2] = Iic_1$ and $[X_3, X_4] = Iic_2$ [18]. This can be generalized to higher dimensional branes. The Four-brane constructed this way will also have membranes (and off course zero-branes) bound to it, and the six-branes will have four-branes and membranes bound to it. We will however in this paper call them a four-brane and a six-brane.

In this paper we analyze this construction. we compute the potential between configurations involving six-branes, four-branes, two-branes and zero-

[1] Another approach can be found in [4]

branes. The potentials are compared with calculations in the type IIa theory of the corresponding configurations. In all cases (as in [15]) we find exact agreements between the m(atrix) description and the type IIa short distance description. Due to the "closeness" of the type IIa configuration to being supesymmetric, we find that the short distance and long distance potentials agree [20], thus enabling the m(atrix) description to describe long distance potentials as well.

It should be noted that the description studied in this paper is only one out of possible constructions for configurations involving four-branes and six-branes.

2 The Calculation

In this section we will calculate the potential between various configurations of gravitons, membranes, four-branes and six-branes in M(atrix) theory. In section (2.1) we describe the interaction between a four-brane and a zero-brane, in section (2.2) the interaction between a four-brane and a membrane parallel to it, in section (2.3) the interaction between two parallel four-branes and in section (2.4) we describe the interaction between a six-brane and a zero-brane.

Let us start with the Lagrangian [5, 1, 21, 22], we take the string length $l_s = 1$, the signature is $(-1, 1 \ldots, 1)$, and $D_t X = \partial_t X - i[A_0, X]$,

$$L = \frac{1}{2g} Tr \left[D_t X_i D_t X^i + 2\theta^T D_t \theta - \frac{1}{2}[X^i, X^j]^2 - 2\theta^T \gamma_i [\theta, X^i] \right]. \tag{1}$$

The supersymmetry transformations are

$$\begin{aligned} \delta X^i &= -2\epsilon^T \gamma^i \theta \\ \delta\theta &= \frac{1}{2}\left[D_t X^i \gamma_i + \frac{1}{2}[X^i, X^j]\gamma_{ij} \right] \epsilon + \epsilon' \\ \delta A_0 &= -2\epsilon^T \theta \end{aligned} \tag{2}$$

We chose to work in the background covariant gauge (the ghost will be called C). We give certain X's some expectation value B and write $X_i = B_i + Y_i$. If one chooses the B_μ such that $B_0 = 0$ and the other B_i solve the equation of motion then we can expand the Lagrangian to quadratic order in the fluctuations around the background fields and find

$$\begin{aligned} L_2 &= \frac{1}{2g} Tr\{(\partial_0 Y_i)^2 - (\partial_0 A_0)^2 - 4i\dot{B}_i[A_0, Y^i] + \frac{1}{2}[B_i, Y_j]^2 + \frac{1}{2}[B_j, Y_i]^2 \quad (3) \\ &+ [B_i, Y^j][Y^i, B^j] + [B_i, Y^i][B_j, Y^j] - [A_0, B_i]^2 + [B_i, B_j][Y^i, Y^j] \\ &+ \partial_0 C^* \partial_0 C + [C^*, B^i][B_i, C] + 2\theta^T \partial_t \theta - 2\theta^T \gamma_i [\theta, B^i]\}. \quad (4) \end{aligned}$$

We take the following form for Y_i and θ.

$$Y_i = \begin{pmatrix} 0 & \phi_i \\ \varphi_i & 0 \end{pmatrix}, \quad \theta = \begin{pmatrix} 0 & \psi \\ \chi & 0 \end{pmatrix}$$

Where in matrix space $\varphi = \phi^\dagger$ and $\chi = \psi^T$.

2.1 Four-brane zero-brane scattering

The background configuration for a zero-brane scattering off a four-brane is [18],

$$B_8 = \begin{pmatrix} P_1 & 0 \\ 0 & 0 \end{pmatrix}, B_9 = \begin{pmatrix} Q_1 & 0 \\ 0 & 0 \end{pmatrix}, B_7 = \begin{pmatrix} P_2 & 0 \\ 0 & 0 \end{pmatrix}, B_6 = \begin{pmatrix} Q_2 & 0 \\ 0 & 0 \end{pmatrix},$$

$$B_5 = \begin{pmatrix} bI & 0 \\ 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} Ivt & 0 \\ 0 & 0 \end{pmatrix}.$$

where $[Q_1, P_1] = ic_1$, $[Q_2, P_2] = ic_2$ and we will soon discuss what are the values of c_1, c_2. The four-brane is thought of as wrapped on a large T^4 of radiuses (R_9, R_8, R_7, R_6) respectively.

The graviton scattering off the four-brane is given to leading order by multiplying the result for the zero-brane, by the number of zero-branes the graviton is made of.

In order to calculate the potential we should compute the mass matrix for ϕ and ψ, and then compute the one loop vacuum energy by evaluating the determinant of the operator $\det(\partial_t^2 + M^2)$. Now if we think of ϕ and ψ as N dimensional vectors (i.e the total number of zero-brane in this bound state N) then we should understand the P_1, Q_1 and P_2, Q_2 matrices as only $N_1 \times N_1$ and $N_2 \times N_2$ matrices, with $N_1 N_2 = N$, as explained in [18]. We will shortly see what this means in term of number of two-brane and zero-branes bounded on the four-brane.

Inserting the above background into equation (4), we find that the mass matrix squared, in the space of $(Y_2 \ldots Y_5, C)$ is proportional to the identity with the proportionality constant being $2H$, and

$$H = P_1^2 + P_2^2 + Q_1^2 + Q_2^2 + Iv^2t^2 + Ib^2. \tag{5}$$

In the space of A_0, Y_1 there are also off diagonal terms of $\pm 4iv$

$$M^2_{A_0 Y_1} = 2 \begin{pmatrix} -H & -2iv \\ 2iv & H \end{pmatrix}$$

In the space of Y_8, Y_9 one has also off diagonal terms $\pm 4ic_1$, and in the space of Y_7, Y_6 one has off diagonal terms $\pm 4ic_2$, thus

$$M^2_{Y_8 Y_9} = 2 \begin{pmatrix} H & -2ic_1 \\ 2ic_1 & H \end{pmatrix}, M^2_{Y_7 Y_6} = 2 \begin{pmatrix} H & -2ic_2 \\ 2ic_2 & H \end{pmatrix}.$$

Evaluating the fermionic terms we find

$$m_f = \gamma_8 P_1 + \gamma_9 Q_1 + \gamma_7 P_2 + \gamma_6 Q_2 + \gamma_1 Ivt + \gamma_5 Ib \tag{6}$$

We now rotate to Euclidean space ($t = i\tau$, $A_0 = -iA_\tau$), and convert the fermionic determinants to a form ($\det(-\partial_\tau^2 + M_f^2)$).

$$M_f^2 = H + Iic_1\gamma_9\gamma_8 + Iic_2\gamma_6\gamma_7 + Iv\gamma_1. \tag{7}$$

This gives for the bosonic determinants two (complex) bosons with $M^2 = 2H$, one with $M^2 = 2H + 4iv$, one with $M^2 = 2H - 4iv$, one with $M^2 = 2H + 4c_1$, one with $M^2 = 2H - 4c_1$, one with $M^2 = 2H + 4c_2$ and one with $M^2 = 2H - 4c_2$. From the fermionic fields we get determinants with M_f^2. Two with $M_f^2 = H + c_1 + c_2 + iv$, two with $M_f^2 = H + c_1 + c_2 - iv$, two with $M_f^2 = H - c_1 - c_2 + iv$, two with $M_f^2 = H - c_1 - c_2 - iv$, two with $M_f^2 = H - c_1 + c_2 + iv$, two with $M_f^2 = H - c_1 + c_2 - iv$, two with $M_f^2 = H + c_1 - c_2 + iv$ and two with $M_f^2 = H + c_1 - c_2 - iv$.

How do the P's and Q's act on ϕ and ψ ?. One can realize these operator on the space of functions of two variables (x, y). Then P_1 can be realized as $-ic_1\partial_x$, Q_1 as x, P_2 as $-ic_2\partial_y$ and Q_2 as y. The spectrum of H is then,

$$H_n = b^2 + v^2t^2 + c_1(2n_1 + 1) + c_2(2n_2 + 1) \tag{8}$$

Define $r_{n_1n_2}^2 = b^2 + c_1(2n_1+1) + c_2(2n_2+1)$ then the phase shift of a zero-brane scattered off the four-brane configuration, to one-loop is

$$\begin{aligned}\delta &= \frac{1}{2}\sum_{n_1n_2}\int \frac{ds}{s} e^{-sr_{n_1n_2}^2} \frac{1}{\sin sv}[2 + 2\cos 2vs + 2\cosh 2sc_1 + 2\cosh 2sc_2 \\ &- 4\cos vs(\cosh(c_1+c_2)s + \cosh(c_1-c_2)s)].\end{aligned} \tag{9}$$

Summing over n_1, n_2 we get

$$\delta = \int \frac{ds}{s} e^{-sb^2} \frac{2 + 2\cos 2vs + 2\cosh 2sc_1 + 2\cosh 2sc_2 - 4\cos vs(\cosh(c_1+c_2)s + \cosh(c_1-c_2)s)}{8\sinh sc_1 \sinh sc_2 \sin sv}. \tag{10}$$

Notice there is a tachyonic instability [23] for $b^2 < |c_1 - c_2|$.

let us compare this with the corresponding string configuration of a four-brane with many two-branes (orthogonally embedded) and many zero-branes, all bounded in a non-threshold bound state. The phase shift of a zero-brane scattering off this bound state was computed in [24], where it was called the $(4-2-2-0)$ bound state (for the classical supergravity solution see [25], and for a T-dual description see [26]). The string configuration is described by a four-brane with a world-volume magnetic field turned on (some configurations were discussed in [28, 27]). ,

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & F_1 & 0 & 0 \\ 0 & -F_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & F_2 \\ 0 & 0 & 0 & -F_2 & 0 \end{pmatrix}.$$

This describes a four-brane with two-branes in the $8,9$ direction, two-branes in the $6,7$ direction and some zero-branes. We choose to take F_1 in the $8,9$ direction and F_2 in the $6,7$ direction. Notice that F_1 describes a two-brane in the four-brane stretched in the $6,7$ direction F_2 a two-brane in the $8,9$ direction. From the coupling of a D-brane to a RR background [19] one can read off the number of embedded two-branes (call them n_1 and n_2). $2\pi R_8 R_9 F_1 = n_1$, $2\pi R_6 R_7 F_2 = n_2$, and the number of zero-branes $N = n_1 n_2$

Define $\tan \pi\epsilon_j = F_j$. Using the same notation as in [24], and $\Theta(\rho) = \Theta(\rho, is)$, the phase shift takes the form,

$$\delta_{IIA} = \frac{1}{2\pi}\int \frac{ds}{s} e^{-b^2 s} B \times J. \tag{11}$$

$$\begin{aligned} B &= \frac{1}{2} f_1^{-4} \Theta_4^{-1}(i\epsilon_1 s)\Theta_4^{-1}(i\epsilon_2 s)\frac{\Theta_1'(0)}{\Theta_1(\nu t)}. \\ J &= \{-f_2^4 \frac{\Theta_2(\nu s)}{\Theta_2(0)}\Theta_3(i\epsilon_1 s)\Theta_3(i\epsilon_2 s) + f_3^4 \Theta_2(i\epsilon_1 s)\Theta_2(i\epsilon_2 s)\frac{\Theta_3(\nu s)}{\Theta_3(0)} \\ &+ f_4^4 \frac{\Theta_4(\nu s)}{\Theta_4(0)}\Theta_1(i\epsilon_1 s)\Theta_1(i\epsilon_2 s)\}. \end{aligned} \tag{12}$$

If F is very large, let $\epsilon_j = \frac{1}{2} - c_j'$ (c' is very small), the phase shift becomes ($\tanh \pi\nu = v$)

$$\delta_{IIA} = \frac{1}{2\pi}\int \frac{ds}{s} e^{-b^2 s} B \times J. \tag{13}$$

$$\begin{aligned} B &= -\frac{1}{2} f_1^{-4} \Theta_1^{-1}(ic_1' s)\Theta_1^{-1}(ic_2' s)\frac{\Theta_1'(0)}{\Theta_1(\nu t)}. \\ J &= \{-f_2^4 \frac{\Theta_2(\nu s)}{\Theta_2(0)}\Theta_2(ic_1' s)\Theta_2(ic_2' s) + f_3^4 \Theta_3(ic_1' s)\Theta_3(ic_2' s)\frac{\Theta_3(\nu s)}{\Theta_3(0)} \\ &- f_4^4 \frac{\Theta_4(\nu s)}{\Theta_4(0)}\Theta_4(ic_1' s)\Theta_4(ic_2' s)\}. \end{aligned} \tag{14}$$

First let us evaluate (14) as if only the massless open string mode would have contributed (i.e very short distances). We find ($\pi c' = c$)

$$B \times J = \pi \frac{2 + 2\cos 2vs + 2\cosh 2sc_1 + 2\cosh 2sc_2 - 8\cos vs(\cosh sc_1 \cosh sc_2)}{4\sinh sc_1 \sinh sc_2 \sin sv} \tag{15}$$

In this limit we get exactly the result from the M(atrix) approach. This is another example that in some sense the M(atrix) description is just another description of a type IIA calculation.

Now we can evaluate c. From the definition of c' one finds $F_j = c_j^{-1}$, we already know the relationship between F and the number of branes, then $c_1 = \frac{2\pi R_8 R_9}{n_1}$ and $c_2 = \frac{2\pi R_6 R_7}{n_2}$. n_1 and n_2 are then the number of zero-branes on the bounded two-branes in the $8, 9$ and $6, 7$ directions respectively. Identifying $N_1 = n_1$ and $N_2 = n_2$, this explains why the P's and Q's were $N_1 \times N_1$ and $N_2 \times N_2$ dimensional matrices. This is consistent with having N_1 and N_2 two-branes in the $6, 7$ and $8, 9$ directions respectively and with having a total of N zero-branes in the bound state.

We can now calculate the long range potential from the M(atrix) calculation and from the string calculation (keeping now the lowest modes of the closed string). Both calculations agree to lowest order in c and v and we find

$$V = -\Gamma(3/2)\frac{2v^2(c_1^2 + c_2^2) + v^4 + (c_1^2 - c_2^2)^2}{8\sqrt{\pi}c_1 c_2}b^{-3}. \tag{16}$$

From equation (16) we see that if $c_1 = c_2$, then there is no force if there is no relative velocity. This is because this configuration then preserves a quarter of the supersymmetry [26, 24], and is a signature of the presence of the four-brane. The agreement of the long distance potentials shows that the four-brane constructed this way has the right tension.

2.2 Four-brane membrane interaction

In this subsection we will compute the velocity dependent potential between a four-brane and a membrane parallel to each other in the m(atrix) theory. The background configuration is

$$B_8 = \begin{pmatrix} P_1 & 0 \\ 0 & P_3 \end{pmatrix}, B_9 = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_3 \end{pmatrix}, B_7 = \begin{pmatrix} P_2 & 0 \\ 0 & 0 \end{pmatrix}, B_6 = \begin{pmatrix} Q_2 & 0 \\ 0 & 0 \end{pmatrix},$$

$$B_5 = \begin{pmatrix} bI & 0 \\ 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} Ivt & 0 \\ 0 & 0 \end{pmatrix},$$

and $c_1 = c_2 = c_3 = c$. Inserting this background to equation (4) we find the mass matrix for the bosons and fermions. Define

$$H = (P_1 + P_3)^2 + (Q_1 - Q_3)^2 + P_2^2 + Q_2^2 + Ib^2 + Iv^2t^2. \tag{17}$$

The mass matrix squared for the bosons in the space $(Y_2 \ldots Y_5, Y_8, Y_9, C)$ is $2IH$. For the other bosons we find

$$M^2_{A_0 Y_1} = 2\begin{pmatrix} -H & -2iv \\ 2iv & H \end{pmatrix}, M^2_{Y_6 Y_7} = 2\begin{pmatrix} H & -2ic_2 \\ 2ic_2 & H \end{pmatrix}.$$

For the fermions one finds

$$m_f = \gamma_8(P_1 + P_3) + \gamma_9(Q_1 - Q_3) + \gamma_7 P_2 + \gamma_6 Q_2 + \gamma_1 Ivt + \gamma_5 Ib. \qquad (18)$$

After rotating to Euclidean space and converting the fermion determinant to the form $\det(-\partial_\tau^2 + M_f^2)$ we find the following: four complex bosons with $M^2 = 2H$, one with $M^2 = 2H + 4iv$, one with $M^2 = 2H - 4iv$, one with $M^2 = 2H + 4c$ and one with $M^2 = 2H - 4c$. For the fermions there is four with $M_f^2 = H + c + iv$, four with $M_f^2 = H + c - iv$, four with $M_f^2 = H - c + iv$ and four with $M_f^2 = H - c - iv$. The P's and Q's can be represented as $Q_1 - Q_3 = x_1$, $Q_1 + Q_3 = y_1$, $Q_2 = x_2$, $P_1 + P_3 = 2ic\partial_{y_1}$, $P_2 = 2ic\partial_{x_2}$ and $P_1 - P_3 = 2ic\partial_{x_1}$. The spectrum of H is then

$$H_{n,x_1,k_1} = b^2 + v^2t^2 + c(2n+1) + 4c^2{k_1}^2 + x_1^2, \qquad (19)$$

and H has a degeneracy which we label by N_- [15]. Evaluating the determinants, summing over n and integrating over (x_1, k_1), the phase shift is

$$\delta = N_- \int \frac{ds}{s} e^{-b^2 s} \frac{4 + 2\cosh 2cs + 2\cos 2vs - 8\cos vs \cosh cs}{16 cs \cosh cs}. \qquad (20)$$

The string description is that of a four-brane with a magnetic field on its world volume (as in section (2.1)), and a two-brane with a magnetic field on its world volume as in [15]. As we took all the c's to be equal we should take all the magnetic fields to be equal and large. The phase shift for the above two-brane when scattered off the above four-brane configuration ($\tan\pi(1/2 - c') = F$, $\tanh \pi\nu = v$)

$$\delta_{IIa} = \frac{L^2(1+F^2)}{2\pi} \int \frac{ds}{s} \frac{e^{-b^2 s}}{4\pi s} B \times J. \qquad (21)$$

Where L^2 is the volume of the two-brane and $B \times J$ is the same as in the case of a zero-brane scattering off a two-brane with a magnetic field on its world volume [15]

$$\begin{aligned} B &= \frac{1}{2} f_1^{-6} (-i\Theta_1)^{-1}(ic's) \frac{\Theta_1'(0)}{\Theta_1(\nu s)}, \\ J &= \{-f_2^6 \frac{\Theta_2(\nu s)}{\Theta_2(0)} \Theta_2(ic's) + f_3^6 \Theta_3(ic's) \frac{\Theta_3(\nu s)}{\Theta_3(0)} \\ &- f_4^6 \frac{\Theta_4(\nu s)}{\Theta_4(0)} \Theta_4(ic's)\}. \end{aligned} \qquad (22)$$

If we now evaluate equation (22) in the limit that the only contribution comes from the lowest modes of the open string, and insert that into equation (21) we find that $\delta_{IIa} = \delta$ (when one identifies $\pi c' = c$, and $N_- = \frac{L^2}{\pi c}$ as in

[15]). Thus the m(atrix) calculation agrees with the short distance string calculation.

Comparing the long distance potentials from the string theory and from the m(atrix) theory we find that to leading order in v and c they agree and give

$$V = -\frac{L^2\Gamma(3/2)(2v^2c^2+c^4+v^4)}{16\pi^{5/2}c^3}b^{-3}. \tag{23}$$

2.3 Four-brane four-brane interaction

In this subsection we will consider the interactions of two of the above four-brane in M(atrix) theory. The background configuration is

$$B_8 = \begin{pmatrix} P_1 & 0 \\ 0 & P_3 \end{pmatrix}, B_9 = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_3 \end{pmatrix}, B_7 = \begin{pmatrix} P_2 & 0 \\ 0 & P_4 \end{pmatrix}, B_6 = \begin{pmatrix} Q_2 & 0 \\ 0 & Q_4 \end{pmatrix},$$

$$B_5 = \begin{pmatrix} bI & 0 \\ 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} Ivt & 0 \\ 0 & 0 \end{pmatrix}.$$

Where we take $c_1 = c_2 = c_3 = c_4 = c$. Define

$$H = (P_1+P_3)^2 + (Q_1-Q_3)^2 + (P_2+P_4)^2 + (Q_2-Q_4)^2 + b^2I + Iv^2t^2. \tag{24}$$

Inserting the background to equation (4) and computing the mass matrix (in Euclidean space), we find for the complex bosons: six with $M^2 = 2H$ one with $M^2 = 2H + 4iv$ and one with $M^2 = 2H - 4iv$. For the fermions: eight with $M_f^2 = H + iv$ and eight with $M_f^2 = H - iv$.

The spectrum of H is continues and there is a degeneracy as in the case of two membranes [15]. We can realize $Q_1 - Q_3 = x_1$, $Q_2 - Q_4 = x_2$, $Q_1 + Q_3 = y_1$ and $Q_2 + Q_4 = y_2$. Then $P_1 + P_3 = -2ic\partial_{y_1}$, $P_1 - P_3 = -2ic\partial_{x_1}$ and similarly for P_2, P_4. The degeneracy will be labeled by N_-.

The phase shift is then

$$\delta = 8N_-^2 \int_{-\infty}^{\infty} dx_1 dx_2 \frac{dk_1}{2\pi}\frac{dk_2}{2\pi} \int \frac{ds}{s} e^{-sr^2_{(x_i,k_i)}} \frac{\sin^4(sv/2)}{sinsv} \tag{25}$$

where

$$r^2_{(x_i,k_i)} = b^2 + 4c^2(k_1^2 + k_2^2) + x_1^2 + x_2^2. \tag{26}$$

Doing the integrals and evaluating the potential one finds

$$V = -\frac{N_-^2\Gamma(3/2)v^4}{32c^2\sqrt{\pi}b^3}. \tag{27}$$

The long range string calculation using [29, 30, 28, 27] gives,

$$V_{IIA} = -\Gamma(3/2)\frac{(1+F^2)^2L^4v^4}{32\pi^{5/2}b^3} \tag{28}$$

Where L^4 is the area of T^4 on which the four-branes are wrapped. Using $F = c^{-1}$ and from [15] $N_- = \frac{L^2}{\pi c}$ we see that the string and M(atrix) calculations agree.

If we would not have taken $c_1 = c_3$ and $c_2 = c_4$ we would have gotten a non zero-force even at $v = 0$. One can also make an anti-four-brane by flipping a sign of one of the P's or Q's.

2.4 Six-brane zero-brane scattering

A six-brane has no bound states with zero-branes. This is because the long range potential is repulsive $\sim \frac{1}{r}$ and the short distance is repulsive $\sim r$. The matrix-theory however describes everything in terms of zero-branes, so one needs to find a configuration with a six-brane that can bind to zero-branes. This can be achieved by adding four-branes and two-branes bounded to the six-brane. In the same spirit as for the four-brane the configuration of a background of a six-brane is

$$B_8 = \begin{pmatrix} P_1 & 0 \\ 0 & 0 \end{pmatrix}, B_9 = \begin{pmatrix} Q_1 & 0 \\ 0 & 0 \end{pmatrix}, B_7 = \begin{pmatrix} P_2 & 0 \\ 0 & 0 \end{pmatrix}, B_6 = \begin{pmatrix} Q_2 & 0 \\ 0 & 0 \end{pmatrix},$$

$$B_5 = \begin{pmatrix} P_3 & 0 \\ 0 & 0 \end{pmatrix}, B_4 = \begin{pmatrix} Q_3 & 0 \\ 0 & 0 \end{pmatrix}, B_3 = \begin{pmatrix} bI & 0 \\ 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} Ivt & 0 \\ 0 & 0 \end{pmatrix}.$$

Here we are going to take $c_1 = c_2 = c_3 = c$, and the six-brane is wrapped on a large T^6 with equal sides of length $2\pi R$. As in the case of the four-brane, if the total number of zero-branes is N then the P, Q matrices should be thought of as $N^{1/3} \times N^{1/3}$ matrices. One substitutes this background into equation (4), and computes the mass matrix. Define

$$H = P_1^2 + P_2^2 + P_3^2 + Q_1^2 + Q_2^2 + Q_3^2 + Ib^2 + Iv^2t^2. \tag{29}$$

In the space of (Y_2, Y_3, C) $M^2 = 2IH$, eventually this sector will not contribute as the ghost will cancel the Y_2, Y_3 contributions. We also find

$$M^2_{A_0Y_1} = 2\begin{pmatrix} -H & -2iv \\ 2iv & H \end{pmatrix} M^2_{Y_8Y_9} = 2\begin{pmatrix} H & -2ic \\ 2ic & H \end{pmatrix}.$$

$$M^2_{Y_7Y_6} = 2\begin{pmatrix} H & -2ic \\ 2ic & H \end{pmatrix}. M^2_{Y_5Y_4} = 2\begin{pmatrix} H & -2ic \\ 2ic & H \end{pmatrix}.$$

For the fermions we find

$$M_f^2 = H + vI\gamma_1 + ic(\gamma_9\gamma_8 + \gamma_6\gamma_7 + \gamma_4\gamma_5). \tag{30}$$

This gives for the complex bosons (in Euclidean space): one with $M^2 = 2H + 4iv$, one with $M^2 = 2H - 4iv$, three with $M^2 = 2H + 4c$ and three with $M^2 = 2H - 4c$.

For the fermions: One with $M_f^2 = H+3c-iv$, one with $M_f^2 = H+3c+iv$, one with $M_f^2 = H-3c-iv$, one with $M_f^2 = H-3c+iv$, three with $M_f^2 = H+c-iv$, three with $M_f^2 = H+c+iv$, three with $M_f^2 = H-c-iv$ and three with $M_f^2 = H-c+iv$.

The P, Q matrices are then realized on the space of functions of three variables (x, y, z). The spectrum of H (similarly to the four-brane case) is

$$H_n = b^2 + v^2t^2 + c(2n_1 + 2n_2 + 2n_3 + 3) \tag{31}$$

The phase shift of a scattered zero-brane off this six-brane is then,

$$\delta = \int \frac{ds}{s} e^{-b^2 s} \frac{2\cos 2vs + 6\cos 2cs - \cos vs(2\cosh 3cs + 6\cosh cs)}{16\sinh^3 cs \sin vs} \tag{32}$$

Notice that in this case there is no tachyonic instability. This would not be the case if $c_1 \neq c_2 \neq c_3$.

We turn know to the corresponding string calculation which is a six-brane with a world volume magnetic field turn on

$$\mathcal{F} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & F_1 & 0 & 0 & 0 & 0 \\ 0 & -F_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & F_2 & 0 & 0 \\ 0 & 0 & 0 & -F_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_3 \\ 0 & 0 & 0 & 0 & 0 & -F_3 & 0 \end{pmatrix}$$

In the M(atrix) configuration we took $c_1 = c_2 = c_3$ so here we take $F_1 = F_2 = F_3 = F$. Define $\tan \pi\epsilon = F$. This configuration describes a six-brane bound with four-branes , two-branes and zero-branes. The phase shift of a zero-brane scattering off this configuration is given at one-loop by,

$$\delta_{IIA} = \frac{1}{2\pi} \int \frac{ds}{s} e^{-b^2 s} B \times J. \tag{33}$$

$$\begin{aligned} B &= \frac{1}{2} f_1^{-2} \Theta_4^{-3}(i\epsilon s) \frac{\Theta_1'(0)}{\Theta_1(\nu t)}. \\ J &= \{-f_2^2 \frac{\Theta_2(\nu s)}{\Theta_2(0)} \Theta_3^3(i\epsilon s) + f_3^2 \Theta_2^3(i\epsilon s) \frac{\Theta_3(\nu s)}{\Theta_3(0)} \\ &- i f_4^2 \frac{\Theta_4(\nu s)}{\Theta_4(0)} \Theta_1^3(i\epsilon s)\}. \end{aligned} \tag{34}$$

If F becomes large it is convenient to define $\epsilon = \frac{1}{2} - c'$, then the phase shift becomes

$$\delta_{IIA} = \frac{1}{2\pi} \int \frac{ds}{s} e^{-b^2 s} B \times J. \tag{35}$$

$$
\begin{aligned}
B &= \frac{1}{2} f_1^{-2} (-i\Theta_1(ic's))^{-3} \frac{\Theta_1'(0)}{\Theta_1(\nu t)}. \\
J &= \{ -f_2^2 \frac{\Theta_2(\nu s)}{\Theta_2(0)} \Theta_2^3(ic's) + f_3^2 \Theta_3^3(ic's) \frac{\Theta_3(\nu s)}{\Theta_3(0)} \\
&- f_4^2 \frac{\Theta_4(\nu s)}{\Theta_4(0)} \Theta_4^3(ic's) \}. \qquad (36)
\end{aligned}
$$

We now follow the same route as in the previous subsection. Expanding equation (36) in the limit when only the lightest open string modes contribute we find ($\pi c' = c$)

$$
B \times J = \pi \frac{6\cosh 2cs + 2\cos 2vs - 8\cos vs \cosh^3 cs}{8\sinh^3 cs \sin vs}. \qquad (37)
$$

Inserting this to the expression for the phase shift and comparing with equation (32) we find that both expressions are the same.

Now $F = \frac{1}{c}$ and $2\pi R^2 = n_4$ the number of four-brane in each direction. Given there are a total of N zero-brane $n_4 = N^{1/3}$, so $c = \frac{2\pi R^2}{N^{1/3}}$. The number of two-brane in each direction is $N^{2/3}$.

The long range potential from the string calculation can be now compared with the long range calculation in the M(atrix) theory. To lowest order in c and v they agree and we find

$$
V = -\Gamma(1/2) \frac{v^4 - 3c^4 + 6v^2c^2}{16\sqrt{\pi} c^3} b^{-1}. \qquad (38)
$$

the repulsive force coming from the term $\sim c^4$ is due to the six-brane. Again the agreement of the long distance potentials shows that the six brane has the right tension.

3 Conclusions

In this paper we explored the construction of four-branes and six-branes in the context of m(atrix) theory. We have computed the potential between membranes and zero-branes, and configurations in m(atrix) theory that include four-branes and six-branes. These results were shown to be identical to a short distance string theory computation in type IIa, of the corresponding configurations. Due to the large number of bounded zero-branes on the six-brane and four-brane, these configuration are very close to being supersymmmetric [15]. Thus the long distance potentials can be reproduced by a short distance calculation involving only the lightest open string modes, so the m(atrix) theory can reproduce the long distance potentials. The agreement of these calculations supports the proposed description of the four-brane and six-brane configurations.

This construction does not give the pure four-brane and six-brane but rather needs more branes to be added in each case as to achieve a state that can bind in a non-threshold bound state with zero-branes. Notice that although the four-brane does have a threshold bound state with zero-branes without the addition of two-branes, the six-brane has no bound states with zero-branes without extra branes added. So while we may hope to be able to describe a pure four-brane, there seem to be an obstacle to describe a pure six-brane.

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